ASGER AABOE

LUNAR AND SOLAR VELOCITIES AND THE LENGTH OF LUNATION INTERVALS IN BABYLONIAN ASTRONOMY

Det Kongelige Danske Videnskabernes Selskab
Matematisk-fysiske Meddelelser **38**, 6

Kommissionær: Munksgaard
København 1971

Synopsis

The fragments of late-Babylonian cuneiform texts published here extend our evidence of the elegant and consistent manner in which account was made of the influence of lunar and solar anomalies upon the variable time intervals between syzygies of the same kind of sun and moon.

Several new functions appear for the first time, most notably two associated with the lengths of six-month intervals.

ACT No. 55 is republished as an appendix. It turns out to be a lunar ephemeris in which the 223 months of the Saros are broken up into twenty intervals: eighteen 12-month intervals, one six-month, and one one-month interval.

PRINTED IN DENMARK
BIANCO LUNOS BOGTRYKKERI A/S

The texts[1] published here, all in the British Museum, and all deriving from unscientific excavations at Babylon some ninety years ago, amplify and extend our evidence for the treatment of the influence of lunar velocity on various interesting time intervals in Babylonian lunar theory according to System A, in the terminology of ACT.[2]

In the course of the last years, a series of texts has luckily come within our ken and control which, in conjunction with the ACT material, has made it possible to understand and appreciate the consistent and elegant manner in which account was made of the effect of lunar and solar anomaly upon the variable time intervals between syzygies of sun and moon.[3]

The problem is, in modern terms, the following: If the velocity of the sun is v_s, that of the moon v_m, and the time of a syzygy t_0, then the length of the subsequent month, Δt, is determined by

$$\int_{t_0}^{t_0 + \Delta t} (v_m - v_s) \mathrm{d}t = 360°,$$

and analogously, if more syzygies are involved.

The Babylonian approach is to separate the effect of the variable lunar velocity and that of the variable solar velocity into two independent, additive terms, for the interval between two consecutive syzygies of the same kind, that is, one true synodic month, is expressed thus:

[1] The texts are published through the courtesy of the Trustees of the British Museum. My visits to the British Museum, as well as part of my subsequent work, were supported by grants from the National Science Foundation. I wish to acknowledge, once again, my debt of gratitude to Dr. Richard Barnett, Keeper, and Dr. Edmond Sollberger, Deputy Keeper, of Western Asiatic Antiquities for their hospitality and helpfulness. Finally, I wish to thank Miss Janice Henderson for her great assistance in the preparation of the reconstructed tables. Moreover, a variety of auxiliary tables concerning B, Φ, and related functions, executed by the Yale Computer according to her very ingenious programs, have greatly facilitated the work with these texts.

[2] For this and similar abbreviations see the Bibliography at the end. I shall adhere, wherever possible, to the terminology of this central work.

[3] See Neugebauer [1], Aaboe [1] and [2]. The present paper is, as it were, the fourth in this sequence of publications of texts, and I shall assume some familiarity with the previous three.

$$\Delta t = 29^{d} + G^{H} + J^{H},$$ [4]

where G depends on lunar velocity, and J on solar velocity, or rather the longitude of the latter syzygy involved.[5] Similarly it was discovered recently that the length of twelve consecutive months is determined by

$$\Delta t = 354^{d} + \Lambda^{H} + Y^{H}$$

where it is Λ that depends on lunar velocity, and Y on longitude.[6]

It is the family of functions to which G and Λ belong that is of primary concern. These functions are derived, not, as one might suspect, from the column representing lunar velocity, but from the related Column Φ through the application of a simple difference principle.

Column Φ, we have learned, measures the length of one Saros, i.e., of 223 consecutive lunations, taking into account only the effect of a variable lunar velocity so that

$$1 \text{ Saros} = 6585^{d} + \Phi^{H}$$

omitting, possibly, a term depending on solar anomaly, or longitude.[7]

The relations between Φ and lunar velocity F, as well as between Φ and G, and Φ and Λ, have been discussed in earlier publications.[8] There is, however, a certain symmetry relation which will be important in the following, and which I should like to derive here, though the derivation is very obvious. First it must be noted that to a certain syzygy is assigned the Φ-value associated with the Saros beginning at that syzygy, while a value of one of the other functions (G, Λ, etc.) is assigned to the syzygy ending the relevant lunation interval.

Figure 1 shows in schematic form the structure of the table generating G from Φ. Here Φ is to appear in its truncated version.[9] The readily justified relation underlying this scheme is

$$G(n+224) = G(n+1) + (\Phi(n+1) - \Phi(n)).$$

The symmetry rule in question may now be derived as follows. The value $\Phi(n)$ measures the Saros bounded by syzygies number n and $n+223$; im-

[4] $1^{d} = 6^{H}$ (large hours) $= 6{,}0^{\circ}$ (time degrees). The *large hour* is a unit introduced for the sake of convenience in the modern literature; the Babylonian unit is the time degree (uš).

[5] There is no distinction between anomalistic and sidereal year in Babylonian astronomy.

[6] cf. Aaboe [2].

[7] cf. Aaboe [1] and [2].

[8] v. d. Waerden [1], p. 148 f., Aaboe [1] and [2].

[9] Whenever a value of Φ exceeds 2,13,20 it is replaced by 2,13,20, and whenever it is smaller than 1,58,31,6,40 it is replaced by 1,58,31,6,40 (see Aaboe [1]).

I	II	III	IV
$\Phi(n)$	$\Phi(n+1)$	$\Phi(n+1)-\Phi(n)$	$G(n+1)$
$\Phi(n+223)$	$\Phi(n+224)$	⋮	$G(n+224)$
⋮	⋮	⋮	⋮

$$G(n+224) = G(n+1) + (\Phi(n+1) - \Phi(n))$$

Fig. 1.

mediately preceding this Saros is the month measured by G(n), and immediately following it is the month measured by $G(n+224)$ as shown in Figure 2.

If we now invert the time axis, the following two changes will take place:

(i) $\Phi(n)$ will now belong to a branch of the opposite kind (ascending or descending) of the zig-zag function, and
(ii) it will be associated with the value G($n+224$) which measures the now immediately preceding month.

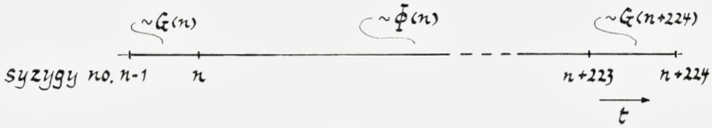


Fig. 2.

Thus, if we take a Φ-value in the first column of the table generating G from Φ, and place it on a branch of the opposite kind, it is to be associated with the G-value in the next line. The same holds for other schemes of the same character.

Texts A and B below doubtless derive from the same tablet, though they do not join physically. Text A consists of ACT No. 207cd joined by three new fragments. My earlier conjecture that ACT No. 207cd once was part of a table generating G from Φ is now confirmed.[10] Further, Text A gives evidence of a hitherto unattested function – I call it Column G' – which, I believe, is G transformed into days and augmented by a constant which I cannot justify to my satisfaction.

In Text C we find Φ related to another new column – I call it Λ' – which is simply Λ increased by a constant for which, once again, I fail to find a satisfactory astronomical motivation.

[10] Aaboe [1], p. 25.

Text D, though fragmentary and ill preserved, presents us with yet another new function – I call it W – belonging to the family of derivates from Φ. I am quite certain that it is associated with a six-month interval, even as Λ is with twelve lunations.

ACT No. 55 is republished and reconstructed as a natural appendix to these new texts. It was principally an awareness of the functions associated with various lunation intervals that made it possible to understand this text. It turns out to be a fragment of an "ephemeris" for conjunctions of sun and moon, probably covering an entire Exeligmos, or triple Saros, with each Saros of 223 months broken up into 18 twelve-month intervals, one one-month interval, and one six-month interval. These various intervals are represented by Λ, G, and a version of W, with their associated corrections for solar anomaly and for change in length of daylight. The agreement between the text and W as reconstructed from Text D is not quite what I had hoped for, though the corresponding correction Z is precisely what was predicted.

A remark about my transcriptions: I usually represent the Babylonian month names by Roman numerals (I = bar, etc.), the two-wedge separation sign, most often used as zero, by ".", and zodiacal signs by their standard sigla. All the texts employ the cursive form of 9. Whenever possible, I outline the preserved surface of the text in the larger reconstructed tables. Photographs of Texts A–D are presented on Plate I.

Text A and B

Text A: B.M. 36699 (80-6-17,431) + 36846 (80-6-17,586) + 37079 (80-6-17, 825) + 37886 (80-6-17,1643)
Text B: B.M. 36908 (80-6-17,649)
Provenance: Babylon (B.M. numbers).
Content: Auxiliary table for Columns Φ, G, G' from lunar System A.
Transcription: Tables 1 and 2.

Description of Text

The fragments which make up Text A form a piece of a tablet with part of the upper edge, but no other edges, preserved. Hand, size of writing, and content indicate that Text B doubtless was part of the same original tablet, but too much clay is missing for a physical join. B. M. 36846 was previously published as ACT No. 207cd.

Horizontal alignment is strictly observed, but the scribe has the habit of occasionally spacing the digits of a short entry so as to fill the entire width of the column.

Identification of obverse and reverse is made on the basis of content and the presence of a horizontal ruling and a blank space near the bottom of the reverse; nothing remains of the expected colophon.

The text appears to be analogous to the one in which Λ is derived from Φ. Thus I expect that when unbroken it had some 60 lines to each side, as does my reconstruction, and four columns, of which only three are still in evidence. The missing column, also giving Φ-values, would be the first of the four.

The preserved parts of Texts A and B are outlined in Tables 1 and 2.

In my transcription of Col. III, I have used "0" to indicate an empty sexagesimal place, though the text has nothing corresponding save, perhaps, a gap between 29 and the next non-zero digit.

Commentary

Of the three preserved columns only two – those giving corresponding values of Φ and G – are in evidence in the fragment published as ACT No. 207cd. From the obverse, and the few traces then visible on the reverse, NEUGEBAUER concluded that the text when unbroken gave a complete list of corresponding values of Φ and G in agreement with the standard conversion table. This conclusion is confirmed by the new fragments, but two new features are now apparent.

First, the first ten lines of Obv. I have entries which are shorter than the normal 6-place sexagesimals of the Φ-values in this column. Whenever endings are preserved they are . . . ,20. It is now clear – as it could not have been at the time of publication of ACT – that the entries all must be 2,13,20, the effective maximum of Φ. This strengthens my earlier suspicion that ACT No. 207cd was a text in which G was generated from Φ by a method analogous to that for deriving Λ from Φ; thus Texts A and B are largely reconstructed according to my earlier proposed scheme for computing G from Φ.[11] I expect that the first preserved column was preceded by another, also giving values of Φ, but corresponding to moments one month earlier, as in the reconstructed table generating G from Φ.

Second, the rejoined fragments give evidence of a hitherto unattested column, Col. III, which I call G'. Col. III is poorly preserved, but since the entries on obverse and reverse are symmetrical, as for G in Col. II, quite a few may be restored with some confidence.

[11] AABOE [1], p. 26.

TABLE 1.

	I	II	III
Obv.	2,13,20	2,40	29 tab
	2,13,20	2,40	29 tab
	2,13,20	2,40,17,[46],40	29,0,3
	2,13,20	2,40,53,20	29,0,9
	2,13,20	2,41,46,40	29,0,18
	2,13,20	2,42,57,46,40	29,0,30
	2,13,20	2,44,26,40	29,0,45
	2,13,20	2,46,13,20	29,1,3
	2,13,20 Text A	2,48,17,46,40	29,1,24
	2,13,20	2,50,40	29,1,48
	2,13,8,8,53,20	2,53,20	29,2,15
	2,12,50,22,13,20	2,56,5,55,33,20	29,2,42,40
	2,12,32,35,33,20	2,58,51,51,6,40	29,3,10,20
	2,12,14,48,53,20	3,1,37,46,40	29,3,38
	2,11,57,2,13,20	3,4,23,42,13,20	29,4,5,40
	2,11,39,15,33,20	3,7,9,37,46,40	29,4,33,20
	2,11,21,28,53,20	3,9,55,33,20	29,5,1
	2,11,3,42,13,20	3,12,41,28,53,20	29,5,28,40
	2,10,45,55,33,20	3,15,27,24,26,40	29,5,56,20
	2,10,28,8,53,20	3,18,13,20	29,6,24
	2,10,10,22,13,20	3,20,59,15,33,20	29,6,51,40
	2,9,52,35,33,20	3,23,45,11,6,40	29,7,19,20
Text B	2,9,34,48,53,20	3,26,31,6,40	29,7,47
	2,9,17,2,13,20	3,29,17,2,13,20	29,8,14,40
	2,8,59,15,33,20	3,32,2,57,46,40	29,8,42,20
	2,8,41,28,53,20	3,34,48,53,20	29,9,10
	2,8,23,42,13,20	3,37,34,48,53,20	29,9,37,40
	2,8,5,55,33,20	3,40,20,44,26,40	29,10,5,20
	2,7,48,8,53,20	3,43,6,40	29,10,33
	2,7,30,22,13,20	3,45,52,35,33,20	29,11,0,40
	2,7,12,35,33,20	3,48,38,31,6,40	29,11,28,20
	2,6,54,48,53,20	3,51,24,26,40	29,11,56
	2,6,37,2,13,20	3,54,10,22,13,20	29,12,23,40
	2,6,19,15,33,20	3,56,56,17,46,40	29,12,51,20
	2,6,1,28,53,20	3,59,42,13,20	29,13,19
	2,5,43,42,13,20	4,2,28,8,53,20	29,13,46,40
	2,5,25,55,33,20	4,5,14,4,26,40	29,14,14,20
	2,5,8,8,53,20	4,8	29,14,42
	2,4,50,22,13,20	4,10,45,55,33,20	29,15,9,40
	2,4,32,35,33,20	4,13,31,51,6,40	29,15,37,20
	2,4,14,48,53,20	4,16,17,46,40	29,16,5
	2,3,57,2,13,20	4,19,3,42,13,20	29,16,32,40
	2,3,39,15,33,20	4,21,49,37,46,40	29,17,0,20
	2,3,21,28,53,20	4,24,35,33,20	29,17,28
	2,3,3,42,13,20	4,27,21,28,53,20	29,17,55,40
	2,2,45,55,33,20	4,30,7,24,26,40	29,18,23,20
	2,2,28,8,53,20	4,32,53,20	29,18,51
	2,2,10,22,13,20	4,35,39,15,33,20	29,19,18,40
	2,1,52,35,33,20	4,38,25,11,6,40	29,19,46,20
	2,1,34,48,53,20	4,41,11,6,40	29,20,14
	2,1,17,2,13,20	4,43,57,2,13,20	29,20,41,40
	2,0,59,15,33,20	4,46,42,57,46,40	29,21,9,20
	2,0,41,28,53,20	4,49,11,6,40	29,21,36,20
	2,0,23,42,13,20	4,51,21,28,53,20	29,22,0,20
	2,0,5,55,33,20	4,53,14,4,26,40	29,22,21,20
	1,59,48,8,53,20	4,54,48,53,20	29,22,39,20
	1,59,30,22,13,20	4,56 (?)	29,22,51,20
	1,59,12,35,33,20	4,56,35,33,20 (?)	29,22,57,20
	1,58,54,48,53,20	4,56,35,33,20 (?)	29,22,57,20
	1,58,37,2,13,20	4,56 (?)	29,22,51,20

TABLE 2.

	I	II	III
Rev. -42.	1,58,37,2,13,20	4,56	29,22,51,20
	1,58,33,42,13,20	4,56	29,22,51,20
-40.	1,58,31,6,40	4,54,48,53,20	29,22,31,20
	1,58,31,6,40	4,53,14,4,26,40	29,22,21,20
	1,58,31,6,40	4,51,21,28,53,20	29,22,0,20
	1,58,31,6,40	4,49,11,6,40	29,21,36,20
	1,58,31,6,40	4,46,42,57,6,40	29,21,9,20
-35.	1,58,48,53,20	4,43,57,2,13,20	29,20,41,40
	1,59,6,40	4,41,11,6,40	29,20,14
	1,59,24,26,40	4,38,25,11,6,40	29,19,46,20
	1,59,42,13,20	4,35,39,15,33,20	29,19,18,40
	2	4,32,53,20	29,18,51
-30.	2,0,17,46,40	4,30,7,24,26,40	29,18,23,20
	2,0,35,33,20	4,27,21,28,53,20	29,17,55,40
	2,0,53,20	4,24,35,33,20	29,17,28
	2,1,11,6,40	4,21,49,37,46,40	29,17,0,20
	2,1,28,53,20	4,19,3,42,13,20	29,16,32,40
-25.	2,1,46,40	4,16,17,46,40	29,16,5
	2,2,4,26,40	4,13,31,51,6,40	29,15,37,20
	2,2,22,13,20	4,10,45,55,33,20	29,15,9,40
	2,2,40	4,8	29,14,42
	2,2,57,46,40	4,5,14,4,26,40	29,14,14,20
-20.	2,3,15,33,20	4,2,28,8,53,20	29,13,46,40
	2,3,33,20	3,59,42,13,20	29,13,19
	2,3,51,6,40	3,56,56,17,46,40	29,12,51,20
	2,4,8,53,20	3,54,10,22,13,20	29,12,23,40
	2,4,26,40	3,51,24,26,40	29,11,56
-15.	2,4,44,26,40	3,48,38,31,6,40	29,11,28,20
	2,5,2,13,20	3,45,52,35,33,20	29,11,0,40
	2,5,20	3,43,6,40	29,10,33
	2,5,37,46,40	3,40,20,44,26,40	29,10,5,20
	2,5,55,33,20	3,37,34,48,53,20	29,9,37,40
-10.	2,6,13,20	3,34,48,53,20	29,9,10
	2,6,31,6,40	3,32,2,57,46,40	29,8,42,20
	2,6,48,53,20	3,29,17,2,13,20	29,8,14,40
	2,7,6,40	3,26,31,6,40	29,7,47
	2,7,24,26,40	3,23,45,11,6,40	29,7,19,20
-5.	2,7,42,13,20	3,20,59,15,33,20	29,6,51,40
	2,8	3,18,13,20	29,6,24
	2,8,17,46,40	3,15,27,24,26,40	29,5,56,20
	2,8,35,33,20	3,12,41,28,53,20	29,5,28,40
	2,8,53,20	3,9,55,33,20	29,5,1
0.	2,9,11,6,40	3,7,9,37,46,40	29,4,33,20
	2,9,28,53,20	3,4,23,42,13,20	29,4,5,40
	2,9,46,40	3,1,37,46,40	29,3,38
	2,10,4,26,40	2,58,51,51,6,40	29,3,10,20
	2,10,22,13,20	2,56,5,55,33,20	29,2,42,40
5.	2,10,40	2,53,20	29,2,15
	2,10,57,46,40	2,50,4[0]	29,1,48
	2,11,15,33,20	2,48,1[7,4]6,40	29,1,24
	2,11,33,20	2,46,13[20]	29,1,3
	2,11,51,6,40	2,44,2[6]40	29,0,45
10.	2,12,8,53,20	2,42,57,4[6,40]	29,0,30
	2,12,26,40	2,41,46,40	29,0,18
	2,12,44,26,40	2,40,5[3,20]	29,0,9
	2,13,2,13,20	2,40,17,46,⌞40⌟	29,0,3
	2,13,20	2,40	29 tab

Text A

Opposite the minimal value of G,

$$G = 2;40^{H},$$

securely restored in Obv. II, 1,2, we find in Col. III the entry "29 tab", i.e., "add 29". The length of the month, disregarding the correction J for solar anomaly, is

$$29^{d} + G^{H},$$

so the meaning of Obv. III, 1,2 is clear, and the units of the entries of Col. III must be *days*.

However, it is trivial that Col. III does not merely give what has to be added to G, for in that case it would be constant. Indeed, the preserved values all fit the conjecture that Col. III represents a new function, G', where

$$G' = 29 + \frac{G - 2;40}{6}. \qquad (1)$$

If this is correct, Col. G' gives the length of a lunation in days, taking into account only the effect of lunar anomaly, but so normed that the minimum of G' is precisely 29^{d}. A justification of this norm escapes me.

A few further comments on G' are in order. First, though the text is broken at or near the end of Col. G', it looks as if the entries in the preserved stretch are limited to three sexagesimal digits. Thus either G' was computed according to (1), and abbreviated to three or four digits, or the parameters of G', as expected from (1), are abbreviated.

I believe the latter is the case. First, if G' itself were abbreviated, then there are instances where the text indicates that the last retained digit is raised even if the first discarded digit is too small according to Babylonian custom. Second, in the $\Phi - \Lambda$ text,[12] Col. *X* is a precedent for abbreviated parameters. Thus I have reconstructed Col. G', first with a constant second difference of

$$\Delta\Delta G' = 0;0,3$$

as an abbreviation of the precise value

$$\Delta\Delta G' = \tfrac{1}{6}\cdot 0;0,17,46,40 = 0;0,2,57,46,40,$$

then with constant first difference

[12] Aaboe [1].

$$\Delta G' = 0;0,27,40$$

as an approximation to

$$\Delta G' = \tfrac{1}{6} \cdot d\Phi = 0;0,27,39,15,33,20,$$

and finally with the second difference in part

$$\Delta\Delta G' = 0;0,3$$

and for the rest

$$\Delta\Delta G' = 0;0,6$$

as required by the structure of G'.

My reconstruction of Col. G' is admittedly quite fanciful.

Text C

Text C: B.M. 36793 (80-6-17,531)
Provenance: Babylon.
Content: Auxiliary table for Columns Φ and Λ' from lunar System A.
Transcription: Table 3.

Description of Text

Text C is a small flake with only one side and part of "upper" edge preserved.

The hand is clear and looks much like that of Texts A and B. I do not believe that this fragment was part of the same tablet as Texts A and B, but it may well be that the tablet it came from belonged to the same series as the one from which Texts A and B were broken.

The preserved writing is in two columns, with horizontal alignment strictly observed.

Commentary

The text gives corresponding values of Φ and of a hitherto unattested function which I call Λ'. To a certain value of Φ corresponds a value of Λ' which obeys, as far as the fragment is preserved, the simple relation

$$\Lambda' = \Lambda + 0;3,24,7,\ \ldots^{H}$$

TABLE 3.

	I	II	[Λ]
1.	[2, 2,42,35]33,20	2,1[8, 6, ... lal]	− 5,42,13,20
	[2, 3, 0,2]2,13,20	1,53,[23, ... tab]	− 1,30,44,26,40
	[2, 3,18,]8,53,20	6,22,[38, ...]	2,58,31, 6,40
	[2, 3,35,5]5,33,20	11, 9,40,[...]	7,45,33,20
5.	[2, 3,53,42] 13,20	16,14,29,[...]	12,50,22,13,20
	[2, 4,11,28] 53,20	21,37, 4 [...]	18,12,57,46,40
	[2, 4,29,15] 33,20	26,57,4[6, ...]	23,35,33,20
	[2, 4,47, 2,13,20]	32,22,[15, ...]	28,58, 8,53,20
	[2, 5, 4,48,53,20]	37,[44,51, ...]	34,20,44,26,40

B.M. 36793

if the entry in Col. II,1 is taken as negative, and the rest as positive. In the transcription I have listed the corresponding values of Λ.

I have no explanation to offer for the constant

$$c = 0;3,24,7,\ldots^{\mathrm{H}}$$

of which no more digits may be reconstructed from the fragmentary Col. II. There is, to be sure, a parameter

3,24[. . .]

in evidence in ACT No. 207ca. The fragmentary passage says:

[fro]m Pisces to Virgo 3,24[. . .]

which would indicate that the constant is a correction tied to the slow arc of solar motion. However, the proper correction of Λ on the slow arc is

$$\mathrm{Y} = 0;21,2,59^{\mathrm{H}}$$

as we learn from the text B.M. 40094.[13]

The only peculiarity of the $\Phi - \Lambda'$ pair that is chosen to be in line 1 is that here the second difference of Λ changes from

$$\Delta\Delta\Lambda = 2 \cdot 0;0,17,46,40 = 0;0,35,33,20$$

to

$$\Delta\Delta\Lambda = 0;0,17,46,40.$$

[13] AABOE [2].

Text D

Text D: B.M. 45930 (SH 81-7-6,366)
Provenance: Babylon (B.M. number).
Content: Auxiliary table for Columns Φ and W from lunar System A.
Transcription: Table 4.

Description of Text

B.M. 45930 is a fragment, $1^1/_2$ inches wide, $2^1/_2$ inches long, with no edges, and only one side, preserved. The surface of the lower half of the preserved side is crumbling badly so that only isolated wedges are visible. The writing is small and in two columns with horizontal alignment observed.

Critical Apparatus

Col. II, l. 5: 1,39,27[...]; 39 might be 29, and 27 could be read 17.
Col. II, l. 10f.: the readings are very uncertain.

Commentary

Text D appears to give corresponding values of Φ and a hitherto unattested function which I call W. The significance of W is that the length of the six months preceding a certain syzygy number n is

$$177^{d} + W(n)^{H}$$

where W(n) is the W-value assigned to syzygy number n, and where account is taken only of the influence of a variable lunar velocity.

Table 4.

	I		II
0.	[2, 3, 15, 33, 20	lal]	[56, 11, 51, 6, 40]
	[2] 2, 57, 46, 40	la[l]	[1, 6, 34, 4, 26, 40]
	[2] 2, 40	lal	[1, 17, 14, 4, 26, 40]
	[2, 3], 22, 13, 20	lal	1, 28, 1[1, 51, 6, 40]
	[2] 2, 4, 26, 40	lal	1, 39, 27, [24, 26, 40]
5.	[2] 1, 46, 40	lal	1, 51, • , 4[4, 26, 40]
	[2] 1, 28, 53, 20	lal	2, 2, 51, 5[1, 6, 40]
	[2] 1, 11, 6, 40	lal	2, 15, • , 4[4, 26, 40]
	[2, 0] 53, 2[0]	lal	[2], 27, 27, [4, 26, 40]
	[2], • , 35, 5[3, 20	la[l]	[2] 40, 11, 5[1, 6, 40]
10.	2, [0, 17, 46, 40	lal]	[2, 53] 14, 4, [26, 40]
	2 [	lal]	[3, 6, 34] 4, [26, 40]
	1, 5[9, 42, 13, 20	lal]	3, [20, 1] 1, 5[1, 6, 40]
	[1, 59, 24, 26, 40	lal]	[3, 34, 7, 24, 26, 40]

B.M. 45930

In table 5 and its continuation, I present the derivation of W from Φ. Using I(n) to denote the entry in Col. I, line n, and similarly, the structure of the table is as follows:

(i) I and II contain Φ-values; I(1) = 2;13,20↑.
(ii) to go from I(n) to I($n+1$) corresponds to an advance of 223 months.
(iii) to go from I(n) to II(n) corresponds to an advance of 6 months.
(iv) III(n) = II(n) − I(n).
(v) IV($n+1$) = IV(n) + III(n).

Column IV gives values of W so that IV(n) is the value of W assigned to the same syzygy as the Φ-value in II(n).

Columns I–III are determined by the rules (i)–(iv), so in addition to rule (v), only an initial value is wanted for the complete determination of Column IV.

I have provided the equivalent of an initial value by fitting plausible endings to the preserved beginnings of the W-values in Text D. The resulting extrema are

$$\max W = 8;35,48,53,20^{\mathrm{H}}$$

and

$$\min W = -0;50,17,2,13,20^{\mathrm{H}}.$$

By taking six monthly values of G symmetrically disposed about the appropriate extremum of G, I obtained the following extrema for the sum of six consecutive values of G:

$$\max \sum_{i=1}^{6} (29^{\mathrm{d}} + \mathrm{G}(i)^{\mathrm{H}}) = 177^{\mathrm{d}} + 8;35,42,57,46,40^{\mathrm{H}}$$

and

$$\min \sum_{i=1}^{6} (29^{\mathrm{d}} + \mathrm{G}(i)^{\mathrm{H}}) = 177^{\mathrm{d}} - 0;50,16,50,22,13,20^{\mathrm{H}}.$$

The agreement is very good, though not perfect. The endings of W, though plausible, cannot, of course, yet be guaranteed. Further, the adjustments of G near its maximum would, *a priori*, make even as close an agreement as this unlikely.

In this reconstructed table for generating W from Φ, I have only given one half, as it were, of the Φ – W scheme. The other half is derived by means of the already justified symmetry rule which we have learnt from the Φ – Λ and Φ – G schemes. It is that if we take a Φ-value in Col. I, line n, place it on the opposite branch, then it is to be assigned the W-value in line $n+1$.

TABLE 5.

	I (Φ)	II (Φ)	III (II − I)	IV (W)
1.	2,13,20↑	2,4,14,4,26,40↓	− 0;9,5,55,33,20	
	2,13,2,13,20↑	2,4,31,51,6,40↓	− 0;8,30,22,13,20	
	2,12,44,26,40↑	2,4,49,37,46,40↓	− 0;7,54,48,53,20	
	2,12,26,40↑	2,5,7,24,26,40↓	− 0;7,19,15,33,20	
5.	2,12,8,53,20↑	2,5,25,11,6,40↓	− 0;6,43,42,13,20	
	2,11,51,6,40↑	2,5,42,57,46,40↓	− 0;6,8,8,53,20	
	2,11,33,20↑	2,6,0,44,26,40↓	− 0;5,32,35,33,20	
	2,11,15,33,20↑	2,6,18,31,6,40↓	− 0;4,57,2,13,20	
	2,10,57,46,40↑	2,6,36,17,46,40↓	− 0;4,21,28,53,20	
10.	2,10,40↑	2,6,54,4,26,40↓	− 0;3,45,55,33,20	
	2,10,22,13,20↑	2,7,11,51,6,40↓	− 0;3,10,22,13,20	
	2,10,4,26,40↑	2,7,29,37,46,40↓	− 0;2,34,48,53,20	
	2,9,46,40↑	2,7,47,24,26,40↓	− 0;1,59,15,33,20	
	2,9,28,53,20↑	2,8,5,11,6,40↓	− 0;1,23,42,13,20	
15.	2,9,11,6,40↑	2,8,22,57,46,40↓	− 0;0,48,8,53,20	
	2,8,53,20↑	2,8,40,44,26,40↓	− 0;0,12,35,33,20	
	2,8,35,33,20↑	2,8,58,31,6,40↓	+ 0;0,22,57,46,40	− 0;50,17,2,13,20
	2,8,17,46,40↑	2,9,16,17,46,40↓	0;0,58,31,6,40	− 0;49,54,4,26,40
	2,8↑	2,9,34,4,26,40↓	0;1,34,4,26,40	− 0;48,55,33,20
20.	2,7,42,13,20↑	2,9,51,51,6,40↓	0;2,9,37,46,40	− 0;47,21,28,53,20
	2,7,24,26,40↑	2,10,9,37,46,40↓	0;2,45,11,6,40	− 0;45,11,51,6,40
	2,7,6,40↑	2,10,27,24,26,40↓	0;3,20,44,26,40	− 0;42,26,40
	2,6,48,53,20↑	2,10,45,11,6,40↓	0;3,56,17,46,40	− 0;39,5,55,33,20
	2,6,31,6,40↑	2,11,2,57,46,40↓	0;4,31,51,6,40	− 0;35,9,37,46,40
25.	2,6,13,20↑	2,11,20,44,26,40↓	0;5,7,24,26,40	− 0;30,37,46,40
	2,5,55,33,20↑	2,11,38,31,6,40↓	0;5,42,57,46,40	− 0;25,30,22,13,20
	2,5,37,46,40↑	2,11,56,17,46,40↓	0;6,18,31,6,40	− 0;19,47,24,26,40
	2,5,20↑	2,12,14,4,26,40↓	0;6,54,4,26,40	− 0;13,28,53,20
	2;5,2,13,20↑	2,12,31,51,6,40↓	0;7,29,37,46,40	− 0;6,34,48,53,20
30.	2,4,44,26,40↑	2,12,49,37,46,40↓	0;8,5,11,6,40	+ 0;0,54,48,53,20
	2,4,26,40↑	2,13,7,24,26,40↓	0;8,40,44,26,40	0;9
	2,4,8,53,20↑	2,13,20 (2,13,25,11,6,40)↓	0;9,11,6,40	0;17,40,44,26,40
	2,3,51,6,40↑	2,13,20 (2,13,42,57,46,40)↓	0;9,28,53,20	0;26,51,51,6,40
	2,3,33,20↑	2,13,20 (2,14,0,44,26,40)↓	0;9,46,40	0;36,20,44,26,40
35.	2,3,15,33,20↑	2,13,20 (2,14,18,31,6,40)↓	0;10,4,26,40	0;46,7,24,26,40
	2,2,57,46,40↑	2,13,20 (2,14,36,17,46,40)↓	0;10,22,13,20	0;56,11,51,6,40
	2,2,40↑	2,13,20 (2,14,54,4,26,40)↓	0;10,40	1;6,34,4,26,40
	2,2,22,13,20↑	2,13,20 (2,15,11,51,6,40)↓	0;10,57,46,40	1;17,14,4,26,40
	2,2,4,26,40↑	2,13,20 (2,15,29,37,46,40)↓	0;11,15,33,20	1;28,11,51,6,40
40.	2,1,46,40↑	2,13,20 (2,15,47,24,26,40)↓	0;11,33,20	1;39,27,24,26,40
	2,1,28,53,20↑	2,13,20 (2,16,5,11,6,40)↓	0;11,51,6,40	1;51,0,44,26,40
	2,1,11,6,40↑	2,13,20 (2,16,22,57,46,40)↓	0;12,8,53,20	2;2,51,51,6,40
	2,0,53,20↑	2,13,20 (2,16,40,44,26,40)↓	0;12,26,40	2;15,0,44,26,40
	2,0,35,33,20↑	2,13,20 (2,16,58,31,6,40)↓	0;12,44,26,40	2;27,27,24,26,40
45.	2,0,17,46,40↑	2,13,20 (2,16,53,20)↑	0;13,2,13,20	2;40,11,51,6,40
	2↑	2,13,20 (2,16,35,33,20)↑	0;13,20	2;53,14,4,26,40
	1,59,42,13,20↑	2,13,20 (2,16,17,46,40)↑	0;13,37,46,40	3;6,34,4,26,40
	1,59,24,26,40↑	2,13,20 (2,16)↑	0;13,55,33,20	3;20,11,51,6,40
	1,59,6,40↑	2,13,20 (2,15,42,13,20)↑	0;14,13,20	3;34,7,24,26,40
50.	1,58,48,53,20↑	2,13,20 (2,15,24,26,40)↑	0;14,31,6,40	3;48,20,44,26,40
	1,58,31,6,40↑	2,13,20 (2,15,6,40)↑	0;14,48,53,20	4;2,51,51,6,40
	1,58,31,6,40 (1,58,13,20)↑	2,13,20 (2,14,48,53,20)↑	0;14,48,53,20	4;17,40,44,26,40
	1,58,31,6,40 (1,57,55,33,20)↑	2,13,20 (2,14,31,6,40)↑	0;14,48,53,20	4;32,29,37,46,40
	1,58,31,6,40 (1,57,58,8,53,20)↓	2,13,20 (2,14,13,20)↑	0;14,48,53,20	4;47,18,31,6,40
55.	1,58,31,6,40 (1,58,15,55,33,20)↓	2,13,20 (2,13,55,33,20)↑	0;14,48,53,20	5;2,7,24,26,40
	1,58,33,42,13,20↓	2,13,20 (2,13,37,46,40)↑	0;14,46,17,46,40	5;16,56,17,46,40

TABLE 5 (continued).

	I	II	III	IV
57.	1,58,51,28,53,20↓	2,13,20↑	0;14,28,31,6,40	5;31,42,35,33,20
	1,59,9,15,33,20↓	2,13,2,13,20↑	0;13,52,57,46,40	5;46,11,6,40
	1,59,27,2,13,20↓	2,12,44,26,40↑	0;13,17,24,26,40	6;0,4,4,26,40
60.	1,59,44,48,53,20↓	2,12,26,40↑	0;12,41,51,6,40	6;13,21,28,53,20
	2,0,2,35,33,20↓	2,12,8,53,20↑	0;12,6,17,46,40	6;26,3,20
	2,0,20,22,13,20↓	2,11,51,6,40↑	0;11,30,44,26,40	6;38,9,37,46,40
	2,0,38,8,53,20↓	2,11,33,20↑	0;10,55,11,6,40	6;49,40,22,13,20
	2,0,55,55,33,20↓	2,11,15,33,20↑	0;10,19,37,46,40	7;0,35,33,20
65.	2,1,13,42,13,20↓	2,10,57,46,40↑	0;9,44,4,26,40	7;10,55,11,6,40
	2,1,31,28,53,20↓	2,10,40↑	0;9,8,31,6,40	7;20,39,15,33,20
	2,1,49,15,33,20↓	2,10,22,13,20↑	0;8,32,57,46,40	7;29,47,46,40
	2,2,7,2,13,20↓	2,10,4,26,40↑	0;7,57,24,26,40	7;38,20,44,26,40
	2,2,24,48,53,20↓	2,9,46,40↑	0;7,21,51,6,40	7;46,18,8,53,20
70.	2,2,42,35,33,20↓	2,9,28,53,20↑	0;6,46,17,46,40	7;53,40
	2,3,0,22,13,20↓	2,9,11,6,40↑	0;6,10,44,26,40	8;0,26,17,46,40
	2,3,18,8,53,20↓	2,8,53,20↑	0;5,35,11,6,40	8;6,37,2,13,20
	2,3,35,55,33,20↓	2,8,35,33,20↑	0;4,59,37,46,40	8;12,12,13,20
	2,3,53,42,13,20↓	2,8,17,46,40↑	0;4,24,4,26,40	8;17,11,51,6,40
75.	2,4,11,28,53,20↓	2,8↑	0;3,48,31,6,40	8;21,35,55,33,20
	2,4,29,15,33,20↓	2,7,42,13,20↑	0;3,12,57,46,40	8;25,24,26,40
	2,4,47,2,13,20↓	2,7,24,26,40↑	0;2,37,24,26,40	8;28,37,24,26,40
	2,5,4,48,53,20↓	2,7,6,40↑	0;2,1,51,6,40	8;31,14,48,53,20
	2,5,22,35,33,20↓	2,6,48,53,20↑	0;1,26,17,46,40	8;33,16,40
80.	2,5,40,22,13,20↓	2,6,31,6,40↑	0;0,50,44,26,40	8;34,42,57,46,40
	2,5,58,8,53,20↓	2,6,13,20↑	+ 0;0,15,11,6,40	8;35,33,42,13,20
	2,6,15,55,33,20↓	2,5,55,33,20↑	− 0;0,20,22,13,20	8;35,48,53,20

Text D contains a fragment of this part of the scheme. The Φ-values are all designated "lal," i.e., they belong to a descending branch. The same Φ-values, but on ascending branches, are found in Col. I, lines 35–48, of Table 5, so the W-values of Text D ought to be the ones in Table 5 one line lower, which indeed they are.

Since it appears that, at least very nearly,

$$W(n) = \sum_{i=n-5}^{n} G(i)$$

we can expect that W must receive a correction for solar anomaly which is simply

$$Z(n) = \sum_{i=n-5}^{n} J(i)$$

where J is the standard correction for solar anomaly to G.

It is not difficult to see that $Z(n)$ will become a continuous, piecewise

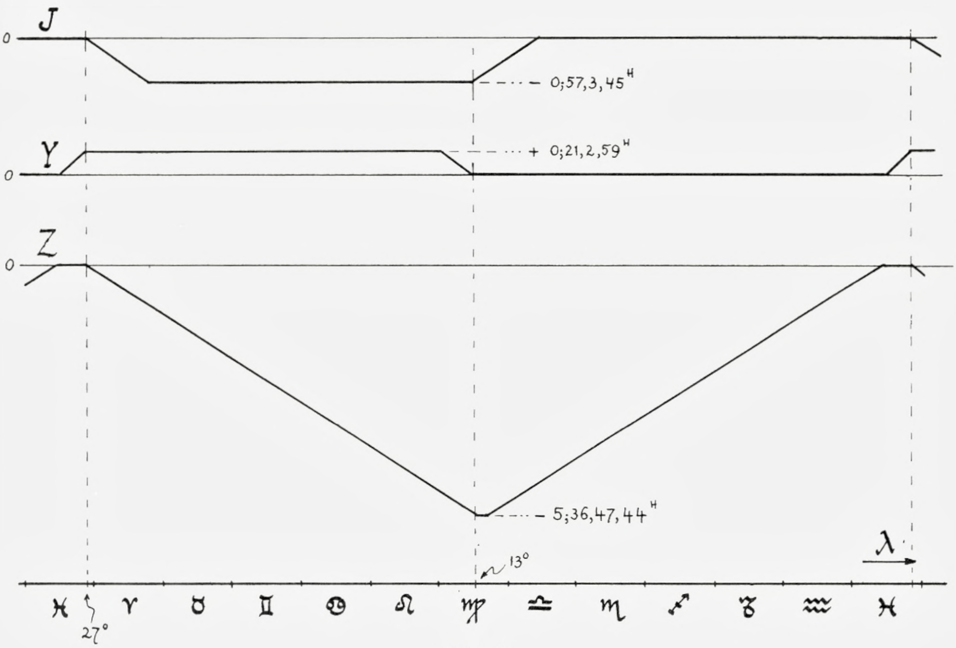


Fig. 3.

linear function of $\lambda(n)$, as is $J(n)$, where $\lambda(n)$ is the longitude of the sun at syzygy number n. Specifically it can be described thus:

Pisces 13° $\leqq \lambda(n) \leqq$ Pisces 27° : $Z(n) = 0^H$
Pisces 27° $\leqq \lambda(n) \leqq$ Virgo 13° : $Z(n)$ decreases linearly to $-5;36,47,44^H$
Virgo 13° $\leqq \lambda(n) \leqq$ Virgo 15;56°: $Z(n) = -5;36,47,44^H$
Virgo 15;56° $\leqq \lambda(n) \leqq$ Pisces 13° : $Z(n)$ increases linearly to 0^H.

A comparison between this correction and the corresponding correction Y to Λ is in order.[14] Y agrees with J only in its differences, as it should, but is subject to the entirely independent condition that it be 0 on the fast arc. This last condition decides the initial value of Λ. The stretches where Y is not constant are very short, so the desire to have Y vanish on one of the constant arcs is very understandable.

The correction Z, however, has very short constant stretches, and with the above definition it is already 0 on one of them. The functions J, Y and Z are compared graphically in Figure 3.

[14] Aaboe [2].

TABLE 6.

	I (Φ)	II (Φ)	III (II - I)	IV (V)
1.	2, 13, 20 ↑	2, 7 ↓	− 0; 6, 40	
	2, 13, 2, 13, 20 ↑	2, 7, 17, 46, 40 ↓	− 0; 5, 44, 26, 40	
	2, 12, 44, 26, 40 ↑	2, 7, 35, 33, 20 ↓	− 0; 5, 8, 53, 20	
	2, 12, 26, 40 ↑	2, 7, 53, 20 ↓	− 0; 4, 33, 20	
5.	2, 12, 8, 53, 20 ↑	2, 8, 11, 6, 40 ↓	− 0; 3, 57, 46, 40	
	2, 11, 51, 6, 40 ↑	2, 8, 28, 53, 20 ↓	− 0; 3, 22, 13, 20	
	2, 11, 33, 20 ↑	2, 8, 46, 40 ↓	− 0; 2, 46, 40	
	2, 11, 15, 33, 20 ↑	2, 9, 4, 26, 40 ↓	− 0; 2, 11, 6, 40	
	2, 10, 57, 46, 40 ↑	2, 9, 22, 13, 20 ↓	− 0; 1, 35, 33, 20	
10.	2, 10, 40 ↑	2, 9, 40 ↓	− 0; 1	
	2, 10, 22, 13, 20 ↑	2, 9, 57, 46, 40 ↓	− 0; 0, 24, 26, 40	
	2, 10, 4, 26, 40 ↑	2, 10, 15, 33, 20 ↓	+ 0; 0, 11, 6, 40	0
	2, 9, 46, 40 ↑	2, 10, 33, 20 ↓	0; 0, 46, 40	0; 0, 11, 6, 40
	2, 9, 28, 53, 20 ↑	2, 10, 51, 6, 40 ↓	0; 1, 22, 13, 20	0; 0, 57, 46, 40
15.	2, 9, 11, 6, 40 ↑	2, 11, 8, 53, 20 ↓	0; 1, 57, 46, 40	0; 2, 20
	2, 8, 53, 20 ↑	2, 11, 26, 40 ↓	0; 2, 33, 20	0; 4, 17, 46, 40
	2, 8, 35, 33, 20 ↑	2, 11, 44, 26, 40 ↓	0; 3, 8, 53, 20	0; 6, 51, 6, 40
	2, 8, 17, 46, 40 ↑	2, 12, 2, 13, 20 ↓	0; 3, 44, 26, 40	0; 10
	2, 8 ↑	2, 12, 20 ↓	0; 4, 20	0; 13, 44, 26, 40
20.	2, 7, 42, 13, 20 ↑	2, 12, 37, 46, 40 ↓	0; 4, 55, 33, 20	0; 18, 4, 26, 40
	2, 7, 24, 26, 40 ↑	2, 12, 55, 33, 20 ↓	0; 5, 31, 6, 40	0; 23
	2, 7, 6, 40 ↑	2, 13, 13, 20 ↓	0; 6, 6, 40	0; 28, 31, 6, 40
	2, 6, 48, 53, 20 ↑	2, 13, 20 (2, 13, 31, 6, 40 ↓)	0; 6, 31, 6, 40	0; 34, 37, 46, 40
	2, 6, 31, 6, 40 ↑	2, 13, 20 (2, 13, 48, 53, 20 ↓)	0; 6, 48, 53, 20	0; 41, 8, 53, 20
25.	2, 6, 13, 20 ↑	2, 13, 20 (2, 14, 6, 40 ↓)	0; 7, 6, 40	0; 47, 57, 46, 40
	2, 5, 55, 33, 20 ↑	2, 13, 20 (2, 14, 24, 26, 40 ↓)	0; 7, 24, 26, 40	0; 55, 4, 26, 40
	2, 5, 37, 46, 40 ↑	2, 13, 20 (2, 14, 42, 13, 20 ↓)	0; 7, 42, 13, 20	1; 2, 28, 53, 20
	2, 5, 20 ↑	2, 13, 20 (2, 15 ↓)	0; 8	1; 10, 11, 6, 40
	2, 5, 2, 13, 20 ↑	2, 13, 20 (2, 15, 17, 46, 40 ↓)	0; 8, 17, 46, 40	1; 18, 11, 6, 40
30.	2, 4, 44, 26, 40 ↑	2, 13, 20 (2, 15, 35, 33, 20 ↓)	0; 8, 35, 33, 20	1; 26, 28, 53, 20
	2, 4, 26, 40 ↑	2, 13, 20 (2, 15, 53, 20 ↓)	0; 8, 53, 20	1; 35, 4, 26, 40
	2, 4, 8, 53, 20 ↑	2, 13, 20 (2, 16, 11, 6, 40 ↓)	0; 9, 11, 6, 40	1; 43, 57, 46, 40
	2, 3, 51, 6, 40 ↑	2, 13, 20 (2, 16, 28, 53, 20 ↓)	0; 9, 28, 53, 20	1; 53, 8, 53, 20
	2, 3, 33, 20 ↑	2, 13, 20 (2, 16, 46, 40 ↓)	0; 9, 46, 40	2; 2, 37, 46, 40
35.	2, 3, 15, 33, 20 ↑	2, 13, 20 (2, 17, 4, 26, 40 ↓)	0; 10, 4, 26, 40	2; 12, 24, 26, 40
	2, 2, 57, 46, 40 ↑	2, 13, 20 (2, 16, 47, 24, 26, 40 ↑)	0; 10, 22, 13, 20	2; 22, 28, 53, 20
	2, 2, 40 ↑	2, 13, 20 (2, 16, 29, 37, 46, 40 ↑)	0; 10, 40	2; 32, 51, 6, 40
	2, 2, 22, 13, 20 ↑	2, 13, 20 (2, 16, 11, 51, 6, 40 ↑)	0; 10, 57, 46, 40	2; 43, 31, 6, 40
	2, 2, 4, 26, 40 ↑	2, 13, 20 (2, 15, 54, 4, 26, 40 ↑)	0; 11, 15, 33, 20	2; 54, 28, 53, 20
40	2, 1, 46, 40 ↑	2, 13, 20 (2, 15, 36, 17, 46, 40 ↑)	0; 11, 33, 20	3; 5, 44, 26, 40
	2, 1, 28, 53, 20 ↑	2, 13, 20 (2, 15, 18, 31, 6, 40 ↑)	0; 11, 51, 6, 40	3; 17, 17, 46, 40
	2, 1, 11, 6, 40 ↑	2, 13, 20 (2, 15, 0, 44, 26, 40 ↑)	0; 12, 8, 53, 20	3; 29, 8, 53, 20
	2, 0, 53, 20 ↑	2, 13, 20 (2, 14, 42, 57, 46, 40 ↑)	0; 12, 26, 40	3; 41, 17, 46, 40
	2, 0, 35, 33, 20 ↑	2, 13, 20 (2, 14, 25, 11, 6, 40 ↑)	0; 12, 44, 26, 40	3; 53, 44, 26, 40
45.	2, 0, 17, 46, 40 ↑	2, 13, 20 (2, 14, 7, 24, 26, 40 ↑)	0; 13, 2, 13, 20	4; 6, 28, 53, 20
	2 ↑	2, 13, 20 (2, 13, 49, 37, 46, 40 ↑)	0; 13, 20	4; 19, 31, 6, 40
	1, 59, 42, 13, 20 ↑	2, 13, 20 (2, 13, 31, 51, 6, 40 ↑)	0; 13, 37, 46, 40	4; 32, 51, 6, 40
	1, 59, 24, 26, 40 ↑	2, 13, 14, 4, 26, 40 ↑	0; 13, 49, 37, 46, 40	4; 46, 28, 53, 20
	1, 59, 6, 40 ↑	2, 12, 56, 17, 46, 40 ↑	0; 13, 49, 37, 46, 40	5; 0, 18, 31, 6, 40
50.	1, 58, 48, 53, 20 ↑	2, 12, 38, 31, 6, 40 ↑	0; 13, 49, 37, 46, 40	5; 14, 8, 8, 53, 20
	1, 58, 31, 6, 40 ↑	2, 12, 20, 44, 26, 40 ↑	0; 13, 49, 37, 46, 40	5; 27, 57, 46, 40
	1, 58, 31, 6, 40 (1, 58, 13, 20 ↑)	2, 12, 2, 57, 46, 40 ↑	0; 13, 31, 51, 6, 40	5; 41, 47, 24, 26, 40
	1, 58, 31, 6, 40 (1, 57, 55, 33, 20 ↑)	2, 11, 45, 11, 6, 40 ↑	0; 13, 14, 4, 26, 40	5; 55, 19, 15, 33, 20
	1, 58, 31, 6, 40 (1, 57, 58, 8, 53, 20 ↓)	2, 11, 27, 24, 26, 40 ↑	0; 12, 56, 17, 46, 40	6; 8, 33, 20
55.	1, 58, 31, 6, 40 (1, 58, 15, 55, 33, 20 ↓)	2, 11, 9, 37, 46, 40 ↑	0; 12, 38, 31, 6, 40	6; 21, 29, 37, 46, 40
	1, 58, 33, 42, 13, 20 ↓	2, 10, 51, 51, 6, 40 ↑	0; 12, 18, 8, 53, 20	6; 34, 8, 8, 53, 20

TABLE 6 (continued).

	I	II	III	IV
57.	1,58,51,28,53,20↓	2,10,34, 4,26,40↑	+ 0;11,42,35,33,20	6;46,26,17,46,40
	1,59, 9,15,33,20↓	2,10,16,17,46,40↑	0;11, 7, 2,13,20	6;57, 8,53,20
	1,59,27, 2,13,20↓	2, 9,58,31, 6,40↑	0;10,31,28,53,20	7; 9,15,55,33,20
60.	1,59,44,48,53,20↓	2, 9,40,44,26,40↑	0; 9,55,55,33,20	7;19,47,24,26,40
	2, 0, 2,35,33,20↓	2, 9,22,57,46,40↑	0; 9,20,22,13,20	7;29,43,20
	2, 0,20,22,13,20↓	2, 9, 5,11, 6,40↑	0; 8,44,48,53,20	7;39, 3,42,13,20
	2, 0,38, 8,53,20↓	2, 8,47,24,26,40↑	0; 8, 9,15,33,20	7;47,48,31, 6,40
	2, 0,55,55,33,20↓	2, 8,29,37,46,40↑	0; 7,33,42,13,20	7;55,57,46,40
65.	2, 1,13,42,13,20↓	2, 8,11,51, 6,40↑	0; 6,58, 8,53,20	8; 3,31,28,53,20
	2, 1,31,28,53,20↓	2, 7,54, 4,26,40↑	0; 6,22,35,33,20	8;10,29,37,46,40
	2, 1,49,15,33,20↓	2, 7,36,17,46,40↑	0; 5,47, 2,13,20	8;16,52,13,20
	2, 2, 7, 2,13,20↓	2, 7,18,31, 6,40↑	0; 5,11,28,53,20	8;22,39,15,33,20
	2, 2,24,48,53,20↓	2, 7, 0,44,26,40↑	0; 4,35,55,33,20	8;27,50,44,26,40
70.	2, 2,42,35,33,20↓	2, 6,42,57,46,40↑	0; 4, 0,22,13,20	8;32,26,40
	2, 3, 0,22,13,20↓	2, 6,25,11, 6,40↑	0; 3,21,48,53,20	8;36,27, 2,13,20
	2, 3,18, 8,53,20↓	2, 6, 7,24,26,40↑	0; 2,41,15,33,20	8;39,51,51, 6,40
	2, 3,35,55,33,20↓	2, 5,49,37,46,40↑	0; 2,13,42,13,20	8;42,41, 6,40
	2, 3,53,42,13,20↓	2, 5,31,51, 6,40↑	0; 1,38, 8,53,20	8;44,54,48,53,20
75.	2, 4,11,28,53,20↓	2, 5,14, 4,26,40↑	0; 1, 2,35,33,20	8;46,32,57,46,40
	2, 4,29,15,33,20↓	2, 4,56,17,46,40↑	+ 0; 0,27, 2,13,20	8;47,35,33,20
	2, 4,47, 2,13,20↓	2, 4,38,31, 6,40↑	− 0; 0, 8,31, 6,40	8;48, 2,35,33,20

The propriety of basing as large reconstructions and conclusions as these on such fragile evidence as Text D may well be questioned, yet I am quite confident of the results set forth above. First, the endings of Φ-values in Col. I of Text D of obvious constant difference 17,46,40 forcibly suggest the canonical column of such values, descending from 2,13,20, present in any scheme for deriving some useful time interval from Φ. Second, the preserved beginnings in Col. II of Text D indicate a constant second difference of 17,46,40. Now two possible interpretations of Col. II come to mind: as a measure of a six-month interval – as W above – or of a five-month interval. The latter possibility is displayed in Table 6 and its continuation. The rules for its construction are like the rules (i)–(v) for the construction of Table 5 except that "6 months" in rule (iii) is replaced by "5 months." Further, I have arbitrarily set the minimum of the fourth column – I call it Col. V – equal to zero. It will be observed that the differences of V in the crucial stretch corresponding to the preserved part of Text D happen to be identical with the differences in W.

Nonetheless, the very excellent agreement between the extrema of W and of sums of six consecutive monthly G-values effectively excludes any other interpretation of Col. II of Text D. However, before Col. V is entirely dismissed I shall discuss certain of its features, for I suspect that such a function may well be found, which is also my excuse for including Table 6.

Indeed, in eclipse texts one proceeds usually in steps of six months, but occasionally in steps of five months. Since the obvious justification for a six-month function W is its usefulness for eclipse computations, a five-month function can equally well be expected.

I find, as above, that the extrema of sums of five consecutive monthly values of G – including the 29 days – are

$$\max \sum_{i=1}^{5} (29^{d} + G(i)^{H}) = 147^{d} + 10;41,31,51,6,40^{H}$$

and

$$\min \sum_{i=1}^{5} (29^{d} + G(i)^{H}) = 147^{d} + 1;53,44,26,40^{H}.$$

The difference between the extrema is

$$8;47,47,24,26,40^{H}$$

which agrees well with the amplitude of V (which is also its maximum in Table 6, since its minimum is 0)

$$\max V = 8;48,2,35,33,20^{H}.$$

If the function V is found I expect its values to be those of Table 6 augmented by about 1;53,..., the large hours of the minimal value of the sum of five consecutive G-values, for the correction for solar anomaly ought to be the sum of five consecutive J-values, in analogy with W.

Appendix

ACT No. 55

ACT No. 55: B.M. 46015 (SH 81-7-6,461).
Provenance: Babylon.
Content: Data concerning selected conjunctions of sun and moon for, at least, Seleucid Era 180, IX to 202, III (132 B.C. ff.).
Transcription: Table 7.

Critical Apparatus

For Obv. Col. I, see ACT p. 116 (it should be noted that since Obv. and Rev. are now joined, I continue the numbering of columns of the obverse to the corresponding columns of the reverse, so that Rev. IV in my terminology is, in fact, the first preserved column of the reverse and identical with NEUGEBAUER's Rev. I).

The few differences between NEUGEBAUER's readings and my reconstruction can all readily be resolved by reference to a photograph of the ill preserved text, e.g., a damaged cursive 9 can be mistaken for a poorly preserved 10, and *vice versa*. Of particular interest is Rev. IV, 7 (NEUGEBAUER's Rev. I, 7): NEUGEBAUER reads 32,39 or 33,39 where the text should have 34,29,9. The bottom of the 4 in 34 is damaged, so the number can be mistaken for a 3, and the 9 in 29 can easily be read as a fuzzy 10, hence NEUGEBAUER's 39.

Description of Text

The text is, as mentioned, poorly preserved with all edges destroyed. Of the surface of the reverse only little remains. When unbroken, the tablet probably had 30 lines to a side.

Commentary

An unprecedented feature of this text – and so one that made its interpretation singularly difficult – is that its columns contain a mixture of different astronomical functions. Hitherto the identification of function and column has justifiably been taken for granted, and it is reflected in the standard terminology of the literature on Babylonian mathematical astronomy where the words column and function are freely interchanged.

It appears that the aim of the tablet is to compute the time of conjunction of sun and moon, not at consecutive syzygies, but at intervals of one, six, or twelve lunations – most frequently twelve.

The functions are separated into columns according to their kind. In Column II, the second preserved column, we find functions which take account of the influence of the variation of lunar velocity: Λ, most frequently, measuring a twelve-month interval, G measuring a one-month interval, and W measuring a six-month interval. These functions are all placed end to end, as it were, so that the value of G in line 4' of the obverse refers to the preceding month, while that of Λ in line 5' refers to the length of preceding 12 months and is derived from a Φ-value belonging to a syzygy twelve lunations later than that in play in line 4', and similarly in all other cases. I have introduced in the leftmost column a tally of syzygies – akin to the Julian days – so normed that the syzygy where Φ_1 assumes the minimal value of Φ is arbitrarily called 1; this is convenient for the tables of Φ and related functions which have come forth from the Yale computer.

Table 7.

	syzygy no.	$[T_1]$	$[\Phi_1]$	$[B_1]$	$[C_1]$	I F(?)	II G, Λ, W'	III J, Y, Z	IV C' etc.	V K etc.
Obv. 1'	1239	3, 0* IX	2, 6, 11, 17, 46, 40 ↓	1;52 ♑	2, 25, 5, 4		(Λ) 54, 27, 6, 9, 26, 40 tab		20, 14 lal	
	1251	3, 1 VIII	2, 11, 33, 53, 20 ↓	20;48 ♐	2, 26, 33, 36		(Λ) 2, 32, . . , 48, 50, 33, 20 tab		44, 16 lal	
	1257	3, 2 II	2, 0, 37, 35, 33, 20 ↑	15;33, 45 ♊	3, 32, 44, 30	[ ]2 tab	(W) 1, . . , 6, 55, 48, 20 tab	2, 39, 23, 40, 30 lal	3, 5, 27 lal	
	1258	III	2, 3, 23, 31, 6, 40 ↑	13;41, 15 ♋	3, 35, 30, 30	[ ]24 [ ]	(G) 4, 24, 16, 32, 35, 33, 20 tab	57, 3, 45 lal	7, 23 lal	
5'	1270	3, 3* III	1, 58, 0, 55, 33, 20 ↑	3;18, 45 ♋	3, 35, 6, 40	[1]1, 12 [ ]	(Λ) 9, 17, 38, 6, 6, 40 tab	21, 2, 59 tab	14, 55 tab	
	1282	3, 4 II	2, 2, 57, 35, 33, 20 ↓	22;56, 15 ♊	3, 33, 43, 30	[ ]8 [ ]	(Λ) [2]10, 2, 5 lal	21, 2[59] tab	41, 35 tab	
	1294	3, 5 II	2, 8, 20, 11, 6, 40 ↓	12;33, 45 ♊	3, 32, 20, 30	[ ]	(Λ) [1, 33, 2]5, 53, 56, 6, 40 tab	21, 2[59] tab	41, 30 tab	
	1306	3, 6* II	2, 13, 42, 46, 40 ↓	2;11, 15 ♊	3, 28, 52, 30	14, 14 lal	(Λ) 3, 10[5]4, 15, 33, 20 tab	21, 2[59 tab	1, 44 tab	
	1318	3, 7 I	2, 15, 4, 15, 33, 20 ↑	21;48, 45 ♉	3, 24, 43, 30		(Λ) 3, 55, 33, 20 tab	21, 2[59] ta[b]	2, 4, 30 tab	
10'	1330	3, 8 I	2, 9, 41, 40 ↑	11;26, 15 ♉	3, 20, 34, 30		(Λ) 3, 37 tab	2[1], 2, [59] tab	2, 4, 30 tab	
	1342	3, 9** I	2, 4, 19, 4, 26, 40 ↑	1;3, 45 ♉	3, 14, 2, 30		(Λ) 2, 3, 27, 2, 27, 13, 20 tab	21, 2, 59 tab	3, 16 tab	
	1354	XII	1, 58, 56, 28, 53, 20 ↑	20;41, 15 ♈	3, 7, 7, 30		(Λ) 25, 53, 19, 46, 6, 40 tab	21, 2, 59 tab	3, 27, 30 tab	
	1366	3, 10 XII	2, 2, 2, 2, 13, 20 ↓	10;18, 45 ♈	3, 0, 12, 30		(Λ) 13, 33, 53, 20 lal	21, 2, 59 tab	3, 57, 30 tab	
	1378	3, 11* XII	2, 7, 24, 37, 46, 40 ↓	29;56, 15 ♓	2, 52, 53, 30		(Λ) 1, 16, 37, 47, 49, 26, 40 tab	21, 2, 59 tab	3, 39, 30 tab	
	1390	3, 12 XI	2, 12, 47, 13, 20 ↓	19;4 ♓	2, 46, 2, 40		(Λ) 2, 54, 11, 30, 30, 33, 20 tab	5, 57, 35, 30 tab	3, 25, 25 tab	
	1402	3, 13 XI	2, 15, 59, 48, 53, 20 ↑	8 ♓	2, 39, 12		(Λ) 3, 55, 33, 20 tab		3, 25, 20 tab	
Rev. 1'	1414	3, 14* XI	2, 10, 37, 13, 20 ↑	26;56 ♒	2, 34, 46, 24		(Λ) 3, 44, 28, 20 tab		2, 42, 48 tab	
	1426	3, 15 X	2, 5, 14, 37, 46, 40 ↑	15;52 ♒	2, 30, 20, 48		(Λ) 2, 20, 15, 8, 33, 53, 20 tab		2, 12, 48 tab	2[22, 27 tab]
	1438	3, 16 X	1, 59, 52, 2, 13, 20 ↑	4;48 ♒	2, 27, 18, 24		(Λ) 42, 41, 25, 52, 46, 40 tab		1, 31, 12 tab	44, 12 tab]
	1450	3, 17* X	2, 1, 6, 28, 53, 20 ↓	23;44 ♑	2, 25, 49, 52		(Λ) 19, 23, 53, 20 lal		4[4]16 tab	1[8, 39 lal]
5'	1462	3, 18 IX	2, 6, 29, 4, 26, 40 ↓	12;40 ♑	2, 24, 21, 20		(Λ) 59, 49, 41, 42, 46, 40 tab		[44,]16 tab	1, . . , 3[3 tab]
	1474	3, 19* IX	2, 11, 51, 40 ↓	1;36 ♑	2, 25, 7, 12		(Λ) 2, 37, 23, 23, 23, 53, 20 tab		2[2, 5]6 lal	2, 3[7 tab]
	1480	3, 20 II	2, 0, 19, 48, 53, 20 ↑	25;41, 15 ♊	3, 34, 5, 30		(W) 6, 49, 10, 55, 33, 20 tab	2, 59, 56, 13, 20 lal	34, 29, 9 lal	3, 14;[45 tab]
	1481	III	2, 3, 5, 44, 26, 40 ↑	23;48, 45 ♋	3, 34, 9, 30		(G) 4, 27, 2, 28, 8, 53, 20 tab	57, 3, 45 lal	2 lal	3, 29, [56 tab]
	1493	3, 21 III	1, 57, 52, 46, 40 ↓	13;26, 15 ♋	3, 35, 32, 30		(Λ) 4, 25, 13, 39, 26, 40 tab	21, 2, 59 tab	41, 30 lal	
10'	1505	3, 22* III	2, 3, 15, 22, 13, 20 ↓	3;3, 45 ♋	3, 35, 4, 30		(Λ) 2, 16, 26, 48, 40 tab	21, 2, 59 tab	14 lal	

ACT No. 55

Column III contains the corresponding functions registering the influence of solar anomaly (or longitude) on these intervals: J correcting G, Z correcting W, or Y correcting Λ.

Column IV gives the corrections due to changing length of daylight so that the entry in Column IV, line n, is

$$\mathrm{IV}(n) = \tfrac{1}{2} \cdot (C(n-1) - \mathrm{C}(n))$$

where C(n) is the length of daylight, in large hours, corresponding to line n.

Column V, which is only partially preserved on the Reverse, gives the equivalent of Column K for G, i.e., the sum of the values in Columns II, III, and IV for the line in question.

The moment of conjunction, in large hours before sunset (denoted šu in the texts), would then arise by continuous subtraction of the values in Column V from an initial value, as usual in lunar texts according to System A; no trace of this column – of the type of Column M – remains. The computation of the *date* of the conjunction – usually 28 or 29 – implies knowledge of the character (full or hollow) of the preceding month or months, which the prediction of the *hour* does not.

A few remarks are in order, and first about the columns which I have reconstructed. The Φ-values were recaptured from recognisable values in the preserved Column II, particularly that of G in Obv. II, 4 and that of Λ in Obv. II, 5.[15] They turned out to be what in ACT are called Φ_1-values, i.e., values of Φ associated with conjunction, rather than opposition, of sun and moon. Relying on the now well-established fact that the function Φ runs uninterruptedly through all the System A lunar texts, I determined the dates in Column T, of course only mod Π_Φ, the period of Φ (6247 lunations or some 505 years), yet the ones given in T – the years in the Seleucid Era – were historically the most likely. That these dates are not only likely, but certain, is established by Columns III and IV, both of which ultimately depend on the longitude of the conjunction, for they are in exact agreement with the longitudes derived from T, taking advantage of the continuability of B_1, the longitude column.

A feature of greatest interest is the appearance in our text of the functions W and Z, the six-month functions, W for the first time in an ephemeris, and Z for the first time anywhere.

I had hopes that the value of W in Obv. II, 3' would serve to determine

[15] For the table converting Φ into G see ACT p. 60; for the $\Phi - \Lambda$ table see Aaboe [2], p. 22.

the endings of W as reconstructed from the Φ–W scheme in Text D above, but I was disappointed. There can be no doubt that NEUGEBAUER's reading

$$W = 1;.,6,55,48,20 \text{ tab}$$

is the best possible from the photograph of the text in its present state, and it is confirmed by Dr. SOLLBERGER who has kindly looked at the original tablet for me. However, my Φ–W scheme given above in the commentary to Text D would have

(i) $$W = +1;0,15,4,51,40$$

corresponding to

$$\Phi = 2;0,37,35,33,20\uparrow,$$

and the sum of the six G-values corresponding to this and the five preceding Φ-values turn out to be (see Table 8)

(ii) $$\sum_{1}^{6} G = 1;0,7,13,20^{H} \text{ (mod } 6^{H}).$$

TABLE 8.

syzygy no.	Φ_1	G_1
1252	2, 8,47,57,46,40 ↓	3;10,45,40,44,26,40
1253	2, 6, 2, 2,13,20 ↓	3;36,34,19,15,33,20
1254	2, 3,16, 6,40 ↓	4; 2,22,57,46,40
1255	2, 0,30,11, 6,40 ↓	4;28,11,36,17,46,40
1256	1,57,51,40 ↑	4;52,33, 1,28,53,20
1257	2, 0,37,35,33,20 ↑	4;49,39,37,46,40
	ΣG =	1; 0, 7,13,20, 0,0

I am not completely sure how this dilemma may be resolved. It is out of the question that the value in the text can be reconciled with what I reconstructed from Text D, for the first four digits of the W-values in this scheme are secure, and (i) is in excess by about 8 in the third significant place. Thus, if a scheme for deriving W directly from Φ was employed, it must have been a variant of the one in evidence in Text D. It seems equally plausible, however, that the author of the text wished to present in this place the sum of six G-values, but did not give the value (ii) for one of two reasons, either that he employed a Φ–G scheme at variance with the standard one,[16]

[16] ACT No. 207ca shows the existence of such variant Φ – G schemes, but little more, though I have recently added a new fragment to it.

or – and this one should not admit unless under duress – that he committed an arithmetical error.

The value in the same line of the correction Z due to solar anomaly, computed according to the rules I predicted above, agrees perfectly with what is preserved in the text.

The next time a six-month interval occurs is between Rev., line 6' and 7', as is clear from the preserved numbers in Col. IV. In Col. III, 7' I have entered the sum (modulo 6^H) of the relevant six G-values.

It, too, is followed immediately by a one-month interval, and then by twelve-month intervals, as is unambiguously clear from the C'-values in Col. IV. Indeed, it is upon these C'-values that a safe bridge can be built from the obverse to the reverse.

It is lucky that this part of the reverse is preserved, for based on it we can perceive yet another feature of the text's structure. It appears that the W-values (and the G-values) occur separated by an interval of 223 lunations, as is immediately seen from the tally numbers. Thus it appears reasonable that the pattern of the text is that the Saros is divided into one six-month interval, one one-month interval, and eighteen twelve-month intervals, so

$$223 \text{ mo.} = (1 \cdot 6 + 1 \cdot 1 + 18 \cdot 12) \text{ mo.}$$

The bottom edge of the tablet must, for reasons of symmetry, have been two or three lines beyond what is preserved of the obverse; a reasonable guess of the size of the unbroken tablet is that it in all contained three Saroi, which would give 30 lines to a side, 20 lines to each Saros.

That this pattern is not an isolated instance is shown by ACT No. 76. The preserved columns of this text, which also concerns conjunctions of sun and moon, are B_1, the longitude column, C_1 which gives length of daylight, and E_1 representing lunar latitude. The text's top and bottom edges are preserved, so we have representatives of all twenty lines of the complete tablet, corresponding to one Saros.

It so happens that ACT No. 55 and No. 76 are very close in time; indeed, if my guess that No. 55 contained three Saroi is correct, then they overlap. It is, however, not so that they contain the same sequence of conjunctions. If No. 55 is continued in the same pattern, only one conjunction in the continuation is also found in No. 76, that at the end of S.E. 208, month I, which happens to be at the end of the six-month interval in No. 76.[17]

[17] ACT No. 75 may derive from a text of similar structure, though only twelve-month intervals are still in evidence in the preserved parts.

It is clear that texts of this sort cannot be intimately associated with solar eclipses, for in that case intervals of five and six months, and combinations thereof, would be relevant, but not, as we have here, a six-month followed by a one-month interval.

Finally a remark about Column I. Neugebauer assumed that it is a Col. F, giving lunar velocity; this still seems the best assumption, for though my attempts at reconstruction have failed we have already so many variants of Col. F that a new one would not be very surprising.

Yale University
New Haven, Connecticut, U.S.A.

Added in Proof

Since my manuscript went to the printer I had occasion to examine the above texts, once again, in the British Museum. As a result, the following critical remarks should be added:

Text C

Col. II, l. 1: 2,1[8,6...lal]; traces check with 8.

Col. II, l. 6: 21,37,4[...]; contrary to my reconstruction, collation shows at least 7, instead of 4.

ACT No. 55

Obv. II, l. 5': 9,17,38,...; 17 looks more like 27.

Rev. III, l. 1'–8': the surface is preserved and is blank where is should be; there are traces of signs in l. 7' and 8', precisely where the writing should begin again. Hints of tab and lal are just visible at the end of Col. II.

Bibliography

AABOE [1]: ASGER AABOE, *Some Lunar Auxiliary Tables and Related Texts from the Late Babylonian Period.* Mat. Fys. Medd. Dan. Vid. Selsk. **36**, no. 12 (1968).

AABOE [2]: ASGER AABOE, *A Computed List of New Moons for 319 B.C. to 316 B.C. from Babylon.* Mat. Fys. Medd. Dan. Vid. Selsk. **37**, no. 3 (1969).

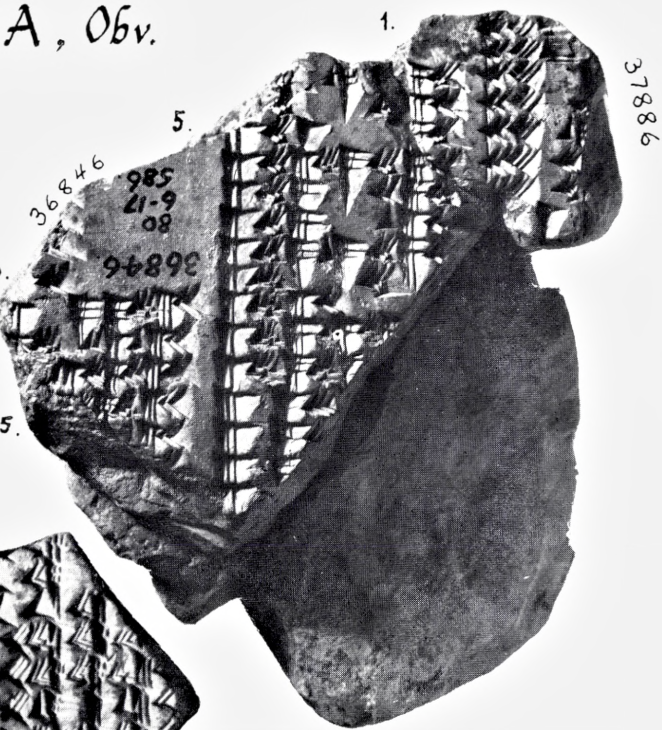
ACT: O. NEUGEBAUER, *Astronomical Cuneiform Texts.* 3 vols. London (Lund Humphries). 1955.

NEUGEBAUER [1]: O. NEUGEBAUER, *"Saros" and Lunar Velocity in Babylonian Astronomy.* Mat. Fys. Medd. Dan. Vid. Selsk. **31**, no. 4 (1957).

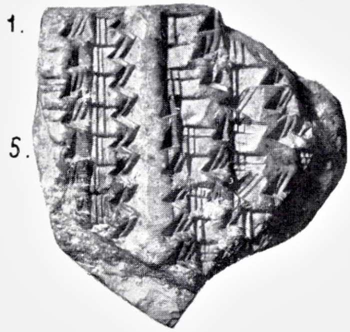
V. D. WAERDEN [1]: B. L. VAN DER WAERDEN, *Anfänge der Astronomie.* Groningen (Noordhoff), 1966.

Indleveret til Selskabet den 16. juni 1970.
Færdig fra trykkeriet den 4. marts 1971.

Text A, Obv.



Text C



Text B

Text A, Rev.

Text D